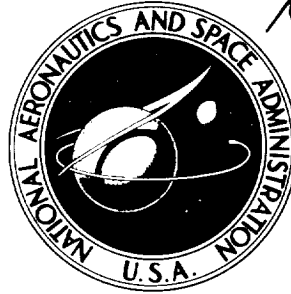


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A METHOD TO OPTIMIZE THE SOLAR CELL POWER SUPPLY FOR INTERPLANETARY SPACECRAFT

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SUMMARY

The purpose of this paper is to present a generalized approach to the development of an optimized solar cell power supply. This method was developed for use on a spin-stabilized spacecraft requiring no control system or orientation mechanism for the solar panels. An optimized power supply is considered to be one that has the proper orientation of solar panels for maximum power output with minimum solar cell weight. In the method described here the area of each solar panel is expressed as a vector in general terms; the solar energy is also expressed as a vector. The power output at any instant is then merely the summation of the dot (inner) product of each area vector and the solar vector.

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INTRODUCTION

The power supply for an interplanetary spacecraft warrants special consideration for two main reasons. First, and most important, the power supply must deliver the necessary power to operate the electrical components of the spacecraft throughout its expected lifetime. Second, the power supply constitutes a large portion of the available useful weight of the spacecraft.

The design of a power supply that will deliver the necessary power for a given trajectory is no great problem. However, there is no assurance that the required power has been delivered with a minimum weight of the solar panels. The expression for the power output at any point in the trajectory will contain two variables, panel area and panel angular orientation. These two variables are determined by the simultaneous evaluation of the expressions for the power output at the two points in the trajectory where the power minima occur.

For a class 1, type 1 trajectory the points of power minima generally occur at launch and at impact. If the points are at other places in the trajectory, reiterations will be necessary to determine the optimum power supply.

This report describes the calculations necessary to determine the minimum area and proper orientation of the solar collectors on a spin-stabilized spacecraft. A typical spacecraft and interplanetary trajectory are shown in Figures 1 and 2. In the calculations the area of an equivalent non-rotating solar collector is determined. (The individual area vectors are described in Appendix A.) This equivalent area is chosen to eliminate the necessity of considering the rotation of the spacecraft when the power equations are evaluated. The rotation of the spacecraft can be neglected because the response time of solar cells is very small when compared with the period of rotation of the spacecraft. A sample calculation for a 130 day flight to Venus is given in Appendix B. For this example a total solar panel area of 50.7 ft² is required to deliver 100 watts.

*A related discussion may be found in "Theoretical Considerations of a Solar Cell Generator on a Satellite," NASA Technical Note D-1904, By Bernard J. Saint-Jean.

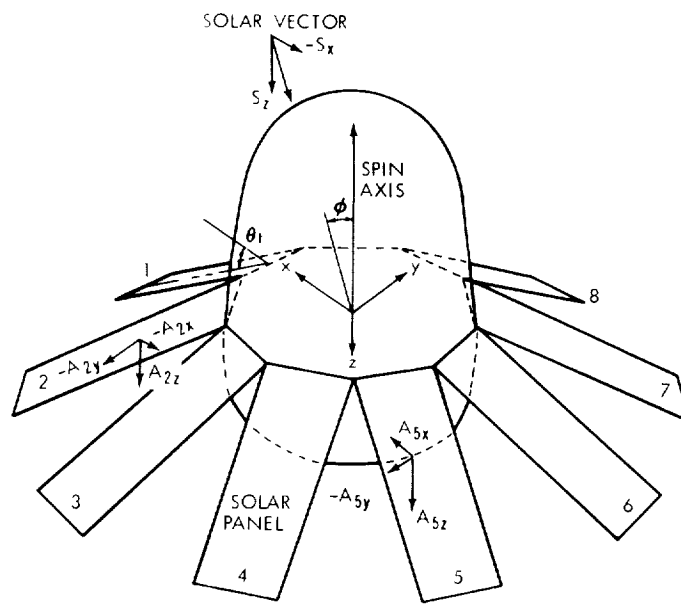


Figure 1—Spacecraft with typical solar panel locations.

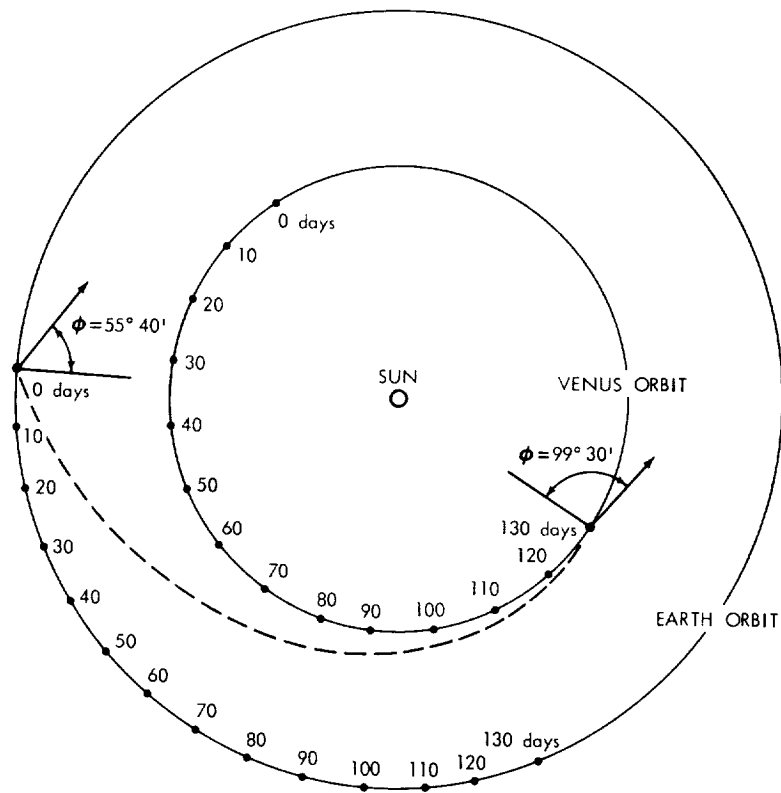


Figure 2—Typical trajectory (type 1, class 1).

INITIAL CONSIDERATIONS

The following symbols are used in this report:

A_n = area vector normal to each panel = $A_{nx} \mathbf{i} + A_{ny} \mathbf{j} + A_{nz} \mathbf{k}$.

d = degradation of solar cells due to temperature (0.6 percent for each degree above 25 °C).

$\mathbf{i}, \mathbf{j}, \mathbf{k}$ = unit vectors in the x, y, z directions of the coordinate system.

P = power output from the solar cell power supply.

p = packing factor (negative value of the percentage of panel area containing active solar cells).

S = solar vector = $S_x \mathbf{i} + S_y \mathbf{j} + S_z \mathbf{k}$.

η = solar cell conversion efficiency — watts (electrical)/watts (solar).

θ_t = angle between each solar panel and the x - y plane of the spacecraft's coordinate system.

ϕ = angle between the spin axis and the spacecraft-sun line.

Figure 3 shows A_n, S, θ_t, ϕ , and the coordinate system.

These assumptions are made:

1. The angle ϕ varies as a function of time throughout the trajectory.
2. The power output from a solar cell varies as the product of the solar constant and the cosine of the angle between the sun rays and the normal to the solar panel.
3. The spacecraft has n attachment points (n solar panels equally spaced around the circumference).
4. The solar panels must be folded and stored during launch, and deployed after injection.

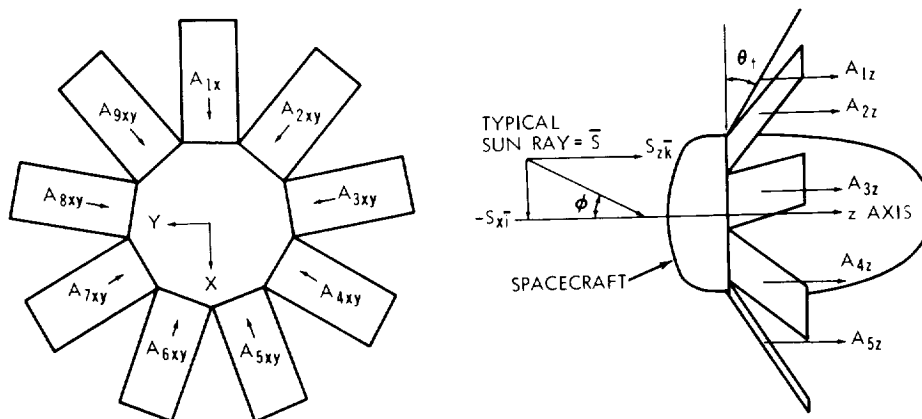


Figure 3—Coordinate system and solar panel orientation.

5. The spacecraft trajectory is known. A sample trajectory, class 1, type 1, is shown in Figure 2:
 - (a) For a type 1 trajectory the spacecraft travels less than half of an orbit around the sun, and for type 2 it travels more than half of an orbit.
 - (b) For a class 1 trajectory the vehicle impacts the target on the first orbital intersection, and for class 2 it impacts on the second orbital intersection.

THE POWER SUPPLY

The general orientation of the solar panels around the spacecraft (Figure 1) was chosen because the power output from a single panel varies over a large range as the angle to the sun rays changes (cosine law). For any single panel, the power output varies from a maximum to zero as the angle of the sun with the normal to the panel varies from 0 to 90 degrees. If the variation of the sun angle is large several panels, each with a different orientation, must be used to insure that the required power is always delivered.

Since the power output from a solar panel is a function of the area of the panel, the solar constant, and the cosine of the angle between the sun and the normal to the panel, the power output readily lends itself to the use of vectors. If the area of each panel is expressed as a vector normal to the surface and the solar constant also is expressed as a vector, the total power output is the dot (inner) product of the area vectors and the solar constant or

$$P = \sum_{n=1}^{n_{max}} \mathbf{S} \cdot \mathbf{A}_n .$$

CHOICE OF COORDINATE SYSTEM

When the trajectory of a spacecraft has been determined, the angle ϕ between the spin axis of the spacecraft and the spacecraft-sun line is known. This suggests a coordinate system centered on the spacecraft with the z axis coinciding with the spin axis. With this choice of a coordinate system, the angle ϕ determines the solar vector.

The x axis is chosen so that the solar vector always lies in the x - z plane (no y component). There is no loss of generality with this choice; it merely reduces the number of computations required. The y axis is chosen perpendicular to the x - z plane, generating a right-hand coordinate system.

SOLAR PANELS REPRESENTED AS VECTORS

With n solar panels spaced around the circumference of a spacecraft, n area vectors can be described. For a smooth power output as the spacecraft spins, the solar panels should be equally spaced around the spacecraft. Each area can be represented as a vector with the following components:

$$A_{nx} \mathbf{i} + A_{ny} \mathbf{j} + A_{nz} \mathbf{k} .$$

Power output from the solar cell power supply varies as the spacecraft spins. The solar panels, spaced symmetrically about the z axis, maintain a constant area component in the z direction. The solar panels are not symmetric with respect to the x or to the y axis at all times. The x and y axes remain fixed in space, and the spacecraft spins in this coordinate system.

The area vectors spinning with the spacecraft in the stationary coordinate system could each be described as a vector rotating around the spin axis (z axis) with the spin rate of the spacecraft. But the response time of solar cells to solar energy is very short (1 millisecond). Therefore, it is equally valid to determine the variation in total effective area exposed to the solar energy as a function of the angular position of the spacecraft around the z axis, and consider the spacecraft to be stationary in the x - y plane at the position where a minimum area is exposed to solar energy.

The point where maximum or minimum effective area is exposed to solar energy can be determined by expressing the n area vectors as previously described with an arbitrary angular displacement ϵ . Then the first derivative of the expression for the total effective area

$$\sum_{n=1}^{n_{max}} A_n$$

is taken and equated to zero, and the points of maximum and minimum effective area are determined. A sample calculation for the maximum and minimum areas for nine solar panels is included in Appendix A.

The area vectors contain the following information:

1. The area and orientation of each solar panel.
2. The solar cell conversion efficiency η .
3. The packing factor p .
4. The degradation due to temperature d .

The area vectors contain a constant A' such that

$$A' = A_n \eta p d ;$$

η and p are known, and d can be determined once the temperature distribution is known.

DESCRIPTION OF THE METHOD

The output from the power supply at any time along the trajectory can be expressed as

$$P = \sum_{n=1}^{n_{max}} S \cdot A_n .$$

In evaluating this summation, all vectors that have a negative power contribution are neglected. The negative sign occurs because the panel is not illuminated by solar energy. Once the trajectory is known, the solar vector can readily be determined. This expression for power output contains two variables, θ_t and A_n . For evaluating these quantities, two equations are required. The equations should be evaluated at the two points where minimum power is delivered from the power supply. For a class 1, type 1 trajectory these points generally occur at launch and impact. Therefore, we will assume these points as the points at which to evaluate the power output in order to solve for the quantities A_n and θ_t . It is also necessary to assume the number of solar panels that are illuminated by solar energy.

By solving for A_n and θ_t , with the use of the original assumptions, a curve for power output as a function of flight time is generated. If the distribution curve confirms the original assumptions of minimum points and the number of active solar cells, the optimum power supply has been determined. If the original assumptions were not correct, the power equations are evaluated at the minimum points determined from the initial power distribution curve. Then it is necessary to again make an assumption on the number of active solar panels. The spin axis orientation at the two power minima and the general orientation of the solar panels must be kept in mind when these assumptions are made.

After the first iteration the assumption concerning the number of active panels is checked. If correct, the design is determined; if not, an additional iteration is required.

CONCLUDING REMARKS

This paper describes a systematic approach to the design of an optimum solar cell power supply. The method greatly reduces the work required to develop a power supply, compared with a conventional trial and error approach.

A more important factor to consider is that this method could be adapted readily to digital computer solutions. In programming the iteration process, the only input information necessary for determining a power supply for any space flight would be:

1. Trajectory.
2. Power requirement.
3. Injection angle.
4. Spacecraft spin axis reorientation, if required.

Further, for any launch window, a family of curves could be generated that would describe the optimum power supply as a function of the injection angle and the trajectory.

Appendix A

Determination of the Minimum and Maximum Areas in the x Direction

For this example assume 9 solar panels, $\eta = 8.5$ percent, $\rho = 90$ percent, and $d = 51.2$ percent. Then

$$A' = A_n (0.085) (0.9) (0.512) = A_n (0.0392) .$$

The area vectors can be expressed as follows (Figure 3):

$$A_1 = A' \sin \theta_t \mathbf{i} + 0 \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_2 = A' \cos 40^\circ \sin \theta_t \mathbf{i} + A' \sin 40^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_3 = A' \cos 80^\circ \sin \theta_t \mathbf{i} + A' \sin 80^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_4 = A' \cos 120^\circ \sin \theta_t \mathbf{i} + A' \sin 120^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_5 = A' \cos 160^\circ \sin \theta_t \mathbf{i} + A' \sin 160^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_6 = A' \cos 200^\circ \sin \theta_t \mathbf{i} + A' \sin 200^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_7 = A' \cos 240^\circ \sin \theta_t \mathbf{i} + A' \sin 240^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_8 = A' \cos 280^\circ \sin \theta_t \mathbf{i} + A' \sin 280^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} ,$$

$$A_9 = A' \cos 320^\circ \sin \theta_t \mathbf{i} + A' \sin 320^\circ \sin \theta_t \mathbf{j} + A' \cos \theta_t \mathbf{k} .$$

The power output is directly proportional to the area of solar panels exposed to solar energy. Therefore, the maximum and minimum areas will be determined. The solar vector has only x and z components; the area in the z direction is constant because of symmetry.

For a power supply consisting of 9 panels, the maximum number of panels illuminated in the x direction is 5. Assume an arbitrary angular displacement between the number 1 panel and the x axis of ϵ . The total area exposed in the x direction is

$$A_T = \sum A_{nx}$$

where $n = 1, 2, 3, 8,$ and 9 ;

$$A_{1x} = A' \cos (\epsilon + 0) \sin \theta_t ,$$

$$A_{2x} = A' \cos (\epsilon + 40^\circ) \sin \theta_t ,$$

$$A_{3x} = A' \cos (\epsilon + 80^\circ) \sin \theta_t ,$$

$$A_{8x} = A' \cos (\epsilon + 280^\circ) \sin \theta_t ,$$

$$A_{9x} = A' \cos (\epsilon + 320^\circ) \sin \theta_t .$$

Expanding the expressions yields:

$$A_{1x} = A' \sin \theta_t (\cos \epsilon \cos 0 - \sin \epsilon \sin 0) ,$$

$$A_{2x} = A' \sin \theta_t (\cos \epsilon \cos 40^\circ - \sin \epsilon \sin 40^\circ) ,$$

$$A_{3x} = A' \sin \theta_t (\cos \epsilon \cos 80^\circ - \sin \epsilon \sin 80^\circ) ,$$

$$A_{8x} = A' \sin \theta_t (\cos \epsilon \cos 280^\circ - \sin \epsilon \sin 280^\circ) ,$$

$$A_{9x} = A' \sin \theta_t (\cos \epsilon \cos 320^\circ - \sin \epsilon \sin 320^\circ) .$$

Therefore,

$$\begin{aligned} \sum A_{nx} &= A' \sin \theta_t [\cos \epsilon (\cos 0 + \cos 40^\circ + \cos 80^\circ + \cos 280^\circ + \cos 320^\circ) \\ &\quad - \sin \epsilon (\sin 0 + \sin 40^\circ + \sin 80^\circ + \sin 280^\circ + \sin 320^\circ)] \end{aligned}$$

$$= A' \sin \theta_t (2.88 \cos \epsilon - 0 \sin \epsilon)$$

$$= 2.88 A' \sin \theta_t \cos \epsilon ;$$

$$\frac{d \sum A_{nx}}{d \epsilon} = -2.88 A' \sin \theta_t \sin \epsilon ,$$

$$\frac{d^2 \sum A_{nx}}{d \epsilon^2} = -2.88 A' \sin \theta_t \cos \epsilon .$$

Setting $d\sum A_{nx}/d\epsilon = 0$ and solving for ϵ yields

$$-2.88 A' \sin \theta_t \sin \epsilon = 0 ,$$

$$\epsilon = 0^\circ; 180^\circ$$

Since $d^2\sum A_{nx}/d\epsilon^2$ is negative the point $\epsilon = 0$ is a maximum.

Because the entire area exposed ($A_x + A_y + A_z$) is constant, and the area exposed in the z direction is also constant (symmetry), the area in the y direction is a minimum when that in the x direction is a maximum. Therefore, to determine the point where minimum area is exposed in the x direction, set $\epsilon = 90^\circ$. Because of the particular panel orientation the 90° rotation is equivalent to a 10° rotation.

The minimum area exposed in the x direction is

$$A_{min} = \sum A_{nx} ;$$

$$A_{1x} = A' \cos 10^\circ \sin \theta_t ,$$

$$A_{2x} = A' \cos 50^\circ \sin \theta_t ,$$

$$A_{3x} = A' \cos 90^\circ \sin \theta_t ,$$

$$A_{8x} = A' \cos 290^\circ \sin \theta_t ,$$

$$A_{9x} = A' \cos 330^\circ \sin \theta_t .$$

Then

$$\begin{aligned} A_{min} &= A' \sin \theta_t (\cos 10^\circ + \cos 50^\circ + \cos 90^\circ + \cos 290^\circ + \cos 330^\circ) \\ &= 2.836 A' \sin \theta_t ; \end{aligned}$$

also

$$A_{max} = 2.879 A' \sin \theta_t .$$

Then the percentage of the variation in the area exposed is

$$\frac{A_{max} - A_{min}}{A_{max}} = \frac{0.044}{2.879} = 1.53 \text{ percent.}$$

For this small variation of 1.5 percent, the panel orientation is arbitrary.

Appendix B

Sample Calculation

Consider a class 1, type 1 trajectory for a space flight to Venus continuously requiring 100 watts of power. The variation of the angle between the spin axis and the sun rays for a typical trajectory is shown in Table B1.

The following steps are required to determine the optimum power supply:

1. Assume two points in the trajectory where a minimum power output should occur.
2. Determine the number of active solar panels at each point.
3. Generate the expressions for the power output at these two points (this yields two equations, each with two unknowns); solve for A' .
4. Evaluate to see whether the assumption of item 2 holds. If it does, proceed; if not, correct and then recalculate A' and θ_t .

Table B1
Variation of the Solar Vector During the Flight.

Time (days)	ϕ	Solar Constant (watt/cm ²)*	Solar Vector*
0	55° 40'	0.135	0.112 i + 0.077 k
10	47°	0.137	0.099 i + 0.092 k
20	38°	0.140	0.087 i + 0.110 k
30	28° 40'	0.147	0.069 i + 0.130 k
40	19° 10'	0.150	0.049 i + 0.140 k
50	9° 10'	0.157	0.023 i + 0.153 k
60	1° 30'	0.167	0 i + 0.167 k
70	12° 40'	0.178	0.038 i + 0.174 k
80	24° 30'	0.192	0.079 i + 0.174 k
90	37° 30'	0.207	0.125 i + 0.163 k
100	51° 20'	0.224	0.174 i + 0.140 k
110	66° 10'	0.241	0.220 i + 0.097 k
120	82° 30'	0.256	0.253 i + 0.033 k
130	99° 30'	0.267	0.263 i + 0.043 k

*The solar constant varies as $1/R^2$, and the solar vector components vary because ϕ varies throughout the trajectory for a spin-stabilized spacecraft.

5. With the correct values for A' and θ_t generate a curve for power output as a function of flight time (see Table B2 and Figure B1).
6. From this power distribution curve evaluate to check item 1. If step 1 checks, the optimum power supply has been determined. If it does not check, evaluate step 3 using the points of power minimum indicated in step 5 and recalculate steps 4, 5, and 6. The optimum power supply has now been determined.
7. Generate the power distribution curve for the optimum power supply design (Table B3 and Figure B1).

Table B2

Values To Be Used in Determining the Power Distribution Curve.

Time (days)	Solar Vector	P_x	P_z	Power Output (watts)
0	$0.112 i + 0.077 k$	55.5	44.5	100.0
10	$0.099 i + 0.092 k$	32.1	74.2	106.3
20	$0.087 i + 0.110 k$	28.2	88.2	116.4
30	$0.069 i + 0.130 k$	-0.0	134.1	134.1
40	$0.049 i + 0.140 k$	0	144.9	144.9
50	$0.023 i + 0.153 k$	0	158.4	158.4
60	$0 i + 0.167 k$	0	172.8	172.8
70	$0.038 i + 0.174 k$	0	180.0	180.0
80	$0.079 i + 0.174 k$	0	180.0	180.0
90	$0.125 i + 0.163 k$	40.6	130.9	171.5
100	$0.174 i + 0.140 k$	56.4	112.7	169.1
110	$0.220 i + 0.097 k$	109.4	55.5	164.9
120	$0.253 i + 0.033 k$	125.9	19.0	144.9
130	$0.263 i + 0.043 k$	130.8	-24.5	106.3

Table B3

Power Distribution.

Time (days)	P_x	P_y	Power Output (watts)
0	53.4	46.5	99.9
10	31.1	77.3	108.4
20	27.3	92.4	119.7
30	0	140.4	140.4
40	0	151.2	151.2
50	0	165.6	165.6
60	0	180.6	180.6
70	0	188.1	188.1
80	24.8	146.3	171.1
90	59.0	98.0	157.0
100	54.0	117.6	172.0
110	105.2	58.5	163.7
120	121.0	20.0	141.0
130	125.9	-26	99.9

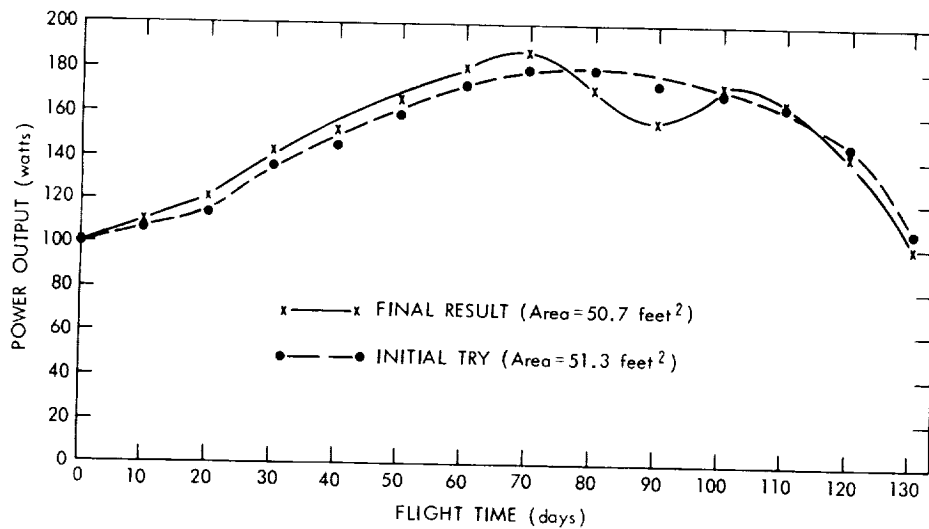


Figure B1 — Power output as a function of time (power required = 100 watts).

From the calculations, at launch the solar constant is a minimum and ϕ is large. This should be a minimum power point. The solar constant varies slowly with time throughout the trajectory. The solar constant is a maximum at the target planet and the angle ϕ is very large. Therefore this should be a minimum also.

Now that the points of minimum have been selected, estimate the number of active solar panels at each point by examining the general configuration of the solar panels. Five should be active at

launch, and three should be active at impact. Now generate the expression for P_L and P_I (power output at launch and impact):

$$\begin{aligned}
 P_L &= \sum S \cdot A_n \\
 &= 0.112 (1 + 2 \cos 40^\circ + 2 \cos 80^\circ) A' \sin \theta_t + 0.077 (5) A' \cos \theta_t \\
 &= (0.112) (2.88) A' \sin \theta_t + (0.077) (5) A' \cos \theta_t \\
 &= 0.323 A' \sin \theta_t + 0.385 A' \cos \theta_t \\
 &= 100 \text{ watts.}
 \end{aligned} \tag{B1}$$

where $n = 1, 2, 3, 8,$ and 9 ;

$$\begin{aligned}
 P_I &= \sum S \cdot A_n \\
 &= 0.263 (1 + 2 \cos 40^\circ) A' \sin \theta_t - 0.043 (3) A' \cos \theta_t \\
 &= 0.665 A' \sin \theta_t - 0.129 A' \cos \theta_t \\
 &= 100 \text{ watts.}
 \end{aligned} \tag{B2}$$

where $n = 1, 2,$ and 9 . From Equation B2

$$A' = \frac{100}{0.665 \sin \theta_t - 0.129 \cos \theta_t}.$$

Substituting into Equation B1 yields:

$$(0.323) (100) \sin \theta_t + (0.385) (100) \cos \theta_t = 100 (0.665 \sin \theta_t - 0.129 \cos \theta_t);$$

$$34.2 \sin \theta_t = 51.4 \cos \theta_t;$$

$$\tan \theta_t = \frac{51.4}{34.2} = 1.5029;$$

$$\theta_t = 56^\circ 22'.$$

Therefore $\sin \theta_t = 0.8326$ and $\cos \theta_t = 0.5539$. Solving for A' yields

$$\begin{aligned} A' &= \frac{100}{0.665 \sin \theta_t - 0.129 \cos \theta_t} \\ &= \frac{100}{0.5536 - 0.0714} \\ &= \frac{100}{0.4822} = 207.4 \end{aligned}$$

Then (from Appendix A)

$$A_n = \frac{A'}{0.0392} = \frac{207.2}{0.0392} = 5290 \text{ cm}^2 ;$$

and

$$A_t = 9 A_n = 47,610 \text{ cm}^2 = 51.3 \text{ ft}^2 .$$

To check the assumptions, generate the dot product of the solar vector and the area vectors, neglecting the negative vector contributions:

$$P = \sum_{n=1}^9 S \cdot A_n .$$

The curve for power output as a function of flight time for this condition, and the values used in determining it, are shown in Table B2 and Figure B1. The contributions from the area vectors (θ_t and A_n now known) at launch and impact are calculated from the following:

$$\begin{aligned} A_1 &= +172.7 \mathbf{i} + 0 \mathbf{j} + 114.9 \mathbf{k} , \\ A_2 &= +132.3 \mathbf{i} + 111.0 \mathbf{j} + 114.9 \mathbf{k} , \\ A_3 &= +30.0 \mathbf{i} + 170.1 \mathbf{j} + 114.9 \mathbf{k} , \\ A_4 &= -86.4 \mathbf{i} + 149.6 \mathbf{j} + 114.9 \mathbf{k} , \\ A_5 &= -162.3 \mathbf{i} + 59.1 \mathbf{j} + 114.9 \mathbf{k} , \\ A_6 &= -162.3 \mathbf{i} + 59.1 \mathbf{j} + 114.9 \mathbf{k} , \\ A_7 &= -86.4 \mathbf{i} + 149.6 \mathbf{j} + 114.9 \mathbf{k} , \\ A_8 &= +30 \mathbf{i} - 170 \mathbf{j} + 114.9 \mathbf{k} , \\ A_9 &= +132.3 \mathbf{i} - 111.0 \mathbf{j} + 114.9 \mathbf{k} , \end{aligned}$$

$$S_L = 0.112 \mathbf{i} + 0 \mathbf{j} + 0.077 \mathbf{k} ,$$

$$S_I = 0.263 \mathbf{i} + 0 \mathbf{j} - 0.043 \mathbf{k} .$$

The launch power contributions are:

$$A_1 = 19.3 + 8.8 = 28.1 ,$$

$$A_2 = 14.8 + 8.8 = 23.6 ,$$

$$A_3 = 3.3 + 8.8 = 12.1 ,$$

$$A_4 = -9.7 + 8.8 = -0.9 ,$$

$$A_5 = -18.2 + 8.8 = -9.4 ,$$

$$A_6 = -18.2 + 8.8 = -9.4 ,$$

$$A_7 = -9.7 + 8.8 = -0.9 ,$$

$$A_8 = +3.3 + 8.8 = +12.1 ,$$

$$A_9 = +14.8 + 8.8 = +23.6 .$$

The impact power contributions are:

$$A_1 = 45.4 - 4.9 = +40.5 ,$$

$$A_2 = 34.8 - 4.9 = +29.9 ,$$

$$A_3 = 7.9 - 4.9 = +3 ,$$

$$A_4 = -22.7 - 4.9 = -27.6 ,$$

$$A_5 = -42.6 - 4.9 = -47.5 ,$$

$$A_6 = -42.6 - 4.9 = -47.5 ,$$

$$A_7 = -22.7 - 4.9 = -27.6 ,$$

$$A_8 = +7.9 - 4.9 = +3 ,$$

$$A_9 = +34.8 - 4.9 = +29.9 .$$

The power contributions from $A_4, 5, 6, 7$ are neglected because the panels are in the shadows, as indicated by the negative sign. The total power contributions are then 99.5 watts in launch and 106.3 watts in impact.

Originally, it was assumed that only three area vectors contribute power at impact. However calculating the dot products indicates five active panels. For all practical purposes the three active panels are correct because of the extremely small contribution from the other two paddles. However,

for this example one reiteration will be performed. Entering the corrections into Equations B1 and B2 gives

$$0.323 A' \sin \theta_t + 0.385 A' \cos \theta_t = 100 ,$$

$$0.757 A' \sin \theta_t - 0.215 A' \cos \theta_t = 100 .$$

Therefore,

$$A' = \frac{100}{0.757 \sin \theta_t - 0.215 \cos \theta_t} ;$$

$$(0.323)(100) \sin \theta_t + (0.385)(100) \cos \theta_t = (0.757)(100) \sin \theta_t - (0.215)(100) \cos \theta_t ;$$

$$43.4 \sin \theta_t = 60.0 \cos \theta_t ;$$

$$\tan \theta_t = \frac{60.0}{43.4} = 1.382 ;$$

$$\theta_t = 54^\circ 7' .$$

Therefore $\sin \theta_t = 0.8102$, $\cos \theta_t = 0.5861$, and

$$A' = \frac{100}{0.757 \sin \theta_t - 0.215 \cos \theta_t}$$

$$= \frac{100}{0.6133 - 0.1260}$$

$$= \frac{100}{0.4873} = 205.2 .$$

Also,

$$A_n = \frac{A'}{0.0392} = \frac{205.2}{0.0392} = 5235 \text{ cm}^2 ;$$

$$A_T = 9 A_n = 47,115 \text{ cm}^2 = 50.7 \text{ ft}^2$$

The area vectors become:

$$A_1 = 116.2 \mathbf{i} + 0 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_2 = 127.3 \mathbf{i} + 106.8 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_3 = 28.9 \mathbf{i} + 163.7 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_4 = -83.1 \mathbf{i} + 143.9 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_5 = -157.1 \mathbf{i} + 56.8 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_6 = -157.1 \mathbf{i} + 56.8 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_7 = -83.1 \mathbf{i} - 143.9 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_8 = +28.9 \mathbf{i} - 163.7 \mathbf{j} + 120.2 \mathbf{k} ,$$

$$A_9 = +127.3 \mathbf{i} - 106.8 \mathbf{j} + 120.2 \mathbf{k} .$$

A comparison of the new values with the first ones shows

$$\theta_{t_1} = 56^\circ 22'; \theta_{t_2} = 54^\circ 7'; A_{n_1} = 5290 \text{ cm}^2; A_{n_2} = 5235 \text{ cm}^2.$$

The angle changed by $2^\circ 15'$ and the area per panel reduced by 55 cm^2 or 1.05 percent. In the example the change was very small, but the power of this approach has been demonstrated. The power output as a function of flight time is shown in Figure B1 and the values used are given in Table B3.

